Algebraic Topology:
Backpaper exam 2004

Attempt all five questions. Each question is worth 20 points. The maximum score is 100

1. Let X be a path-connected, locally path-connected, semi-locally simply-connected space with $A \subset X$ a connected subset with $i : A \to X$ the inclusion map. Let $p : \tilde{X} \to X$ be the universal covering. Show that $p^{-1}(A)$ is connected if and only if for all $a \in A$, $i_* : \pi_1(A, a) \to \pi_1(X, a)$ is surjective.

2. Let G be a finitely generated group and let $k \in \mathbb{N}$. Show that there are at most finitely many subgroups of G of index k.

3. Let $L(p, q)$ denote the quotient of $S^3 \subset \mathbb{C}^2$ by the action of the cyclic group generated by $(z_1, z_2) \mapsto (\zeta z_1, \zeta^q z_2)$, where $\zeta = e^{2\pi i/p}$. Compute the homology of $L(p, q)$.

4. Let $f : S^1 \to S^1$ be a map of degree two. Show that f has a fixed point.

5. Let F be an orientable surface of genus 2. Compute the cohomology ring of F.